The development of interesting connections between the radiuses of circles that are inscribed in or by triangles, and the discovery of unique features, with algebraic manipulations and dynamic exploration.

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The discovery of connections between the radius of a circle inscribing a triangle, a circle inscribed by a triangle, the radiuses of the inscribed circles outside the triangle (tangent to the outside of one side and to the continuation of the other two sides), and the connection between them and the length of the triangle sides, and some parts of the triangle, is a fascinating topic, which can be investigated by raising hypotheses and investigating them dynamically with the computerized technological tool, and also with formulaic mathematical proofs, with a use of various mathematical tools, assisted by algebraic manipulations.

Since Leonhard Euler discovered in 1765 that there are 9 points in a triangle that intercept with a single circle, and the discovery in 1822 by Karl Wilhelm Feuerbach that this circle is externally tangent to the triangle's three excircles, mathematicians have been interested in finding connections between radiuses and different sections.

As part of an advanced course for math teachers on the topic of the combination of fields of mathematics, the students were asked to investigate the question: **Do the Euler circle and the triangle's incircle intersect, are tangent to each other, or unconnected?**

At first, the issue was investigated dynamically by the Geogebra software, and when a positive answer was given, the students had to prove it mathematically, using known formulae derived from many sources.

Subsequent to that, the student were asked to prove the following connections:

(1) $\frac{1}{r} = \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c}$

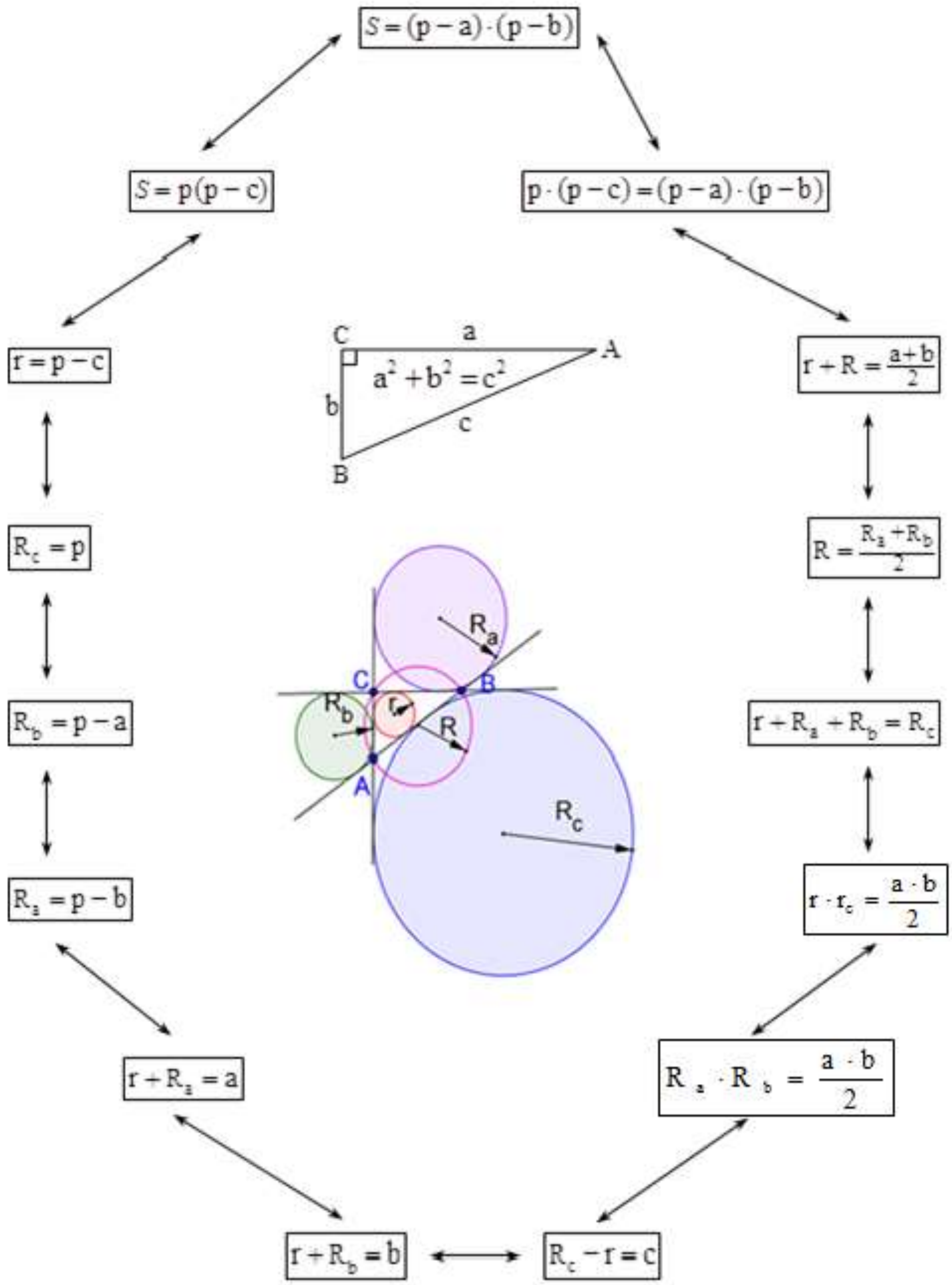
(2) $R_a + R_b + R_c \geq 9r$

(3) $\frac{P}{S_{\Delta ABC}} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$

(4) $R_a + R_b + R_c = 4R + r$

The triangle ΔABC has sides a, b and c. Its incircle has a radius of r. R_a, R_b and R_c are the radiuses of the external circles tangent to the sides a, b and c respectively, and tangent to the extension of the triangle sides. P is the half the circumference of the triangle ΔABC. h_a, h_b, and h_c are their heights.

The significance of relationships among the various elements of the right triangle can be understood from the following schema:



Another task investigated was:

Formula of sum of the distances from the center of the triangle circumcircle to the sides of the triangle.

Triangle ΔABC, which is not obtuse, is inscribed in a circle (O, R). Lines are drawn from the center of the circle O to the sides of the triangle: h_1, h_2, and h_3.

This is true: $h_1 + h_2 + h_3 = R + r$, when r is the radius of the triangle inscribed circle.

Does this characteristic apply for every triangle?

The following points were studied methodically as part of the activity:

- The level of assimilation of the technical tool among the students are the probability of reuse.
- The students' ability to combine tools and different mathematical fields in order to achieve proof and carry out research.
- The level of contribution of research tasks to the students' mathematical knowledge.